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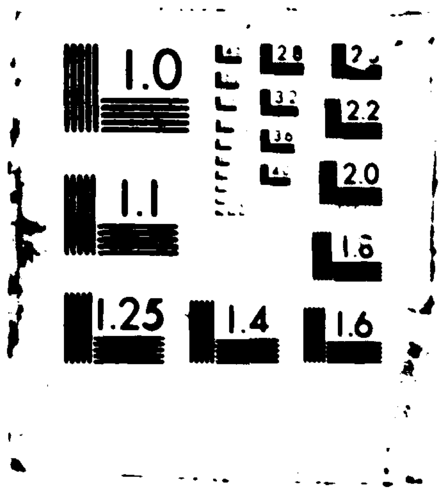
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Analysis of Structures with Rotating, Flexible Substructures Applied to Rotorcraft Aeroelasticity in GRASP

Dewey H. Hodges*
Georgia Institute of Technology, Atlanta, Georgia
A. Stewart Hopkins† and Donald L. Kunz†
U. S. Army Aviation Research and Technology Activity (AVSCOM)
Ames Research Center, Moffett Field, California

Abstract

The initial version of the General Rotorcraft Aeromechanical Stability Program (GRASP) was developed for analysis of rotorcraft in steady, axial flight and ground contact conditions. In these flight regimes, the material continua of the rotorcraft may experience deformations which are independent of time. GRASP can obtain this steady-state solution and can solve the eigenproblem associated with perturbations about such a steady-state solution. GRASP is the first program implementing a new methodology for dynamic analysis of structures, parts of which may be experiencing discrete motion relative to other parts. Application of this new methodology to GRASP, including substructuring, frames of reference, nodes, finite elements and constraints, is described in the paper. GRASP combines features usually found in traditional finite element programs with those usually found in the multibody programs used for spacecraft analysis. The structure is decomposed into a hierarchy of substructures, and discrete relative motion between substructures is treated exactly. Deformation of continua is treated by the finite element method. The library of finite elements includes a powerful nonlinear beam element that incorporates aeroelastic effects based on a simple nonlinear, aerodynamic theory with unsteady induced inflow. The analytical bases for the aeroelastic beam element and the screw constraint are presented, and the important role of geometric stiffness in the formulation is illustrated.

Nomenclature

b = basis vector
C = direction cosine array or damping matrix
e = unit vector or eliminated
F = current frame of reference
G = geometric
g = general constraint relationship function
I = inertial frame of reference
K = stiffness matrix
L = linear operator
M = mass matrix
N = number of
Q = generalized force
q = generalized coordinate
R = position
r = retained
S = superordinate (parent) frame of reference
Δ = the identity matrix
δ = variation of
δW = virtual work
$\delta\psi$ = virtual rotation
ϵ = Levi-Civita symbol
θ = magnitude of Euler rotation or components of a small rotation
ϕ = Euler-Rodrigues parameters

Conventions

$\underline{a}$ = the Gibbsian vector named a
$\grave{a}$ = the unit vector a
$\bar{a}$ = the steady-state value of a
$\dot{a}$ = the perturbation value of a
$\tilde{a}$ = the associated skew symmetric matrix for the vector a
a' = a after steady-state deformation
a'' = a after steady-state deformation and perturbation
a^{bc} = the a associated with b relative to c
a^T = the transpose of the matrix a
a_{b_i} = the ith component of the vector a in the b basis
a_i = the ith component of the vector a or the ith instance of the quantity a
$a \times b$ = vector cross product of a and b

Introduction

Over the years, research analyses that treat various aspects of rotorcraft aeroelastic stability have evolved from very simple models[1] that were used to gain insight into the physical phenomena, into more complex models[2-4] endeavoring to accurately represent realistic rotorcraft configurations. At the same time, large simulation programs[5-8] which include many sophisticated features not present in the research codes, were being developed to solve a variety of rotorcraft problems. These programs made significant contributions to the understanding of rotorcraft modeling. As discussed in Ref. 9, however, they were not designed with rotorcraft stability as their primary application, nor do they possess the depth, generality, and modeling flexibility needed to handle the full range of rotorcraft stability problems.

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* Professor, School of Aerospace Engineering. Associate Fellow AIAA. Member AHS.

† Research Scientist, Rotorcraft Dynamics Division, Aeroflightdynamics Directorate. Member AIAA, AHS.

It is against this background that the General Rotorcraft Aeromechanical Stability Program (GRASP) was developed. The initial version of GRASP was designed specifically for the purpose of calculating the aeromechanical stability of rotorcraft in axial flight and ground contact. Configurations or conditions that would result in periodic or otherwise time-dependent motion cannot currently be accommodated. GRASP is designed to provide a library of solution methods; the initial version provides a steady-state solution and an eigensolution. The steady-state solution is one in which the deformations within each material continuum of the structure remain constant, although there may be discrete motions between the material continua. The eigenproblem solution is one in which the deformation within each material continuum is a constant, reference deformation (typically the steady-state solution) plus a perturbation. Eigenvalues and eigenvectors can be obtained from the linearized, perturbation equations. Expansion of GRASP capabilities to other flight regimes can be accomplished by adding solutions to the library. A description of the solution procedures lies outside the scope of this paper. However, a detailed discussion may be found in Ref. 10.

GRASP is the first program which has been written utilizing the new methodology described in a companion paper.[11] Research in spacecraft dynamics provided the basis for this methodology, which incorporates body flexibility with the large discrete motions previously available with multibody programs.[12] The concept underlying the methodology is that a structure is modeled as composed of substructures which may undergo discrete motions relative to one another. The substructures themselves are deformable elastic continua which are represented by finite element models in GRASP. Thus, through the use of libraries of nodes, elements, and constraints, GRASP provides the powerful modeling capabilities normally available with the finite element method. In addition GRASP has the capability for allowing large discrete motions between substructures.

The primary intent of this paper is to describe the implementation of the new methodology in GRASP. In so doing, some of the more important features of the theoretical basis for GRASP are described. A complete presentation of the equations for all aspects of the GRASP analysis is beyond the scope of this paper but may be found in the GRASP Theoretical Manual.[10]

The first section of this paper describes the GRASP implementation of the decomposition of the structure into substructures. The second and third sections, respectively, explain the concepts of substructure reference frames and nodes. The fourth section describes the implementation of finite elements in GRASP. Three finite elements are currently in the GRASP library: the aeroelastic beam, the air mass, and the rigid-body mass. A description of the aeroelastic beam is presented as an example of the way a finite element is implemented in GRASP. The fifth section outlines the development of constraints. As an example of how the constraint equations are derived, a kinematical development for the screw constraint is presented. Through this example, the role that geometric stiffness plays in the constraint equations is also demonstrated.

Substructuring

The first step of the methodology is the decomposition of the structure into substructures such that discrete motion occurs only between substructures (*e.g.*, the rotor and fuselage of a rotorcraft must be in separate substructures). In GRASP, substructuring is extended to a hierarchical collection of substructures. That is, the structure is decomposed into substructures, which in turn are decomposed into (sub-)substructures (a sub-substructure will just be called a substructure) and so on. The process is continued until all the lowest level substructures consist of a single finite element. The user's input to GRASP uses a slightly different terminology. The elements are referred to as element-type subsystems and all of the other substructures, including the complete structure, are referred to as system-type subsystems. Discrete motion is prohibited within element-type subsystems but may be specified within a system-type subsystem.

There is a great deal of flexibility in the way a structure can be decomposed into a hierarchy of substructures, providing a powerful organizational tool for modeling. Besides isolating discrete motions, some of the uses for substructuring are to separate physical components, to provide logical separation by function, to isolate changing components for parametric studies, and to allow natural coordinatization of components.

The hierarchical organization of the substructures is represented in GRASP as a tree information structure. The root of the tree is created internally by GRASP and corresponds to the degrees of freedom of the complete structure after applying constraints. The only child of the root is the complete structure (a system-type subsystem). From that point, the tree branches out into the substructures which branch out into substructures until, at the most detailed level, called the leaves of the tree, each substructure contains a single finite element (element-type subsystem). The GRASP user conveys the position of each substructure in the tree by including the name of its parent subsystem in the definition of the subsystem containing the substructure. It should be emphasized that a substructure is a subset of its parent, not a branch off of it. Thus, the topology of the structure is *not* restricted to a tree configuration.

Within GRASP, instead of associating the term substructure with its physical embodiment, the term may be used to identify a collection of abstractions or data structures associated with the model of the substructure. From that point of view, a substructure is thought of as being composed of a frame of reference, a set of nodes, a set of substructures, a set of finite elements (which are viewed internally as just more substructures), and the set of constraints which can be used to extract the independent generalized coordinates for the substructure from all of the generalized coordinates for the substructure.

Each substructure is also associated with a state vector, a residual vector, and a set of mass, damping, and stiffness matrices. Starting at the lowest level (a substructure consisting of a single finite element), the elements of these arrays are associated with the substructure's generalized coordinates. This includes the generalized coordinates for the element's nodes, for the element's nodeless variables, and, since

an element is a substructure and thus has a frame of reference, for the element's frame generalized coordinates. After applying the constraints for the substructure (element), only the independent generalized coordinates remain. The independent generalized coordinates for the substructure become part of the complete set of generalized coordinates in the parent structure. This process continues with each parent structure containing the generalized coordinates of its children. Finally, at the root, the elements of the arrays are associated with every independent generalized coordinate in the whole structure (after applying the set of constraints associated with the whole structure).

The general concept of hierarchical substructuring is that everything that belongs to a substructure is just a subset of what belongs to the parent. However, in GRASP, even though the node in a child is logically part of the parent, only the independent generalized coordinates from a node are available in the parent. The node itself is not available in the parent. This poses a difficulty when, for instance, constraining a node in a child to a node in the parent. It is desirable to have the constraint defined within one substructure for a variety of programming reasons and because the constraint is naturally written in one frame of reference. In GRASP, this is resolved by creating another node in the parent, associated with the same physical region of the material continuum as the node in the child. Since there are now two nodes and two sets of generalized coordinates associated with the same set of physical degrees of freedom, a constraint is required. Such a node promotion constraint (there are also node demotion constraints) requires that the two nodes behave identically. In GRASP, these constraints are entirely invisible from the user's point of view. GRASP automatically generates nodes and constraints for its internal use. The user merely specifies that nodes in different substructures are to be constrained together.

Reference Frames

In GRASP, the overall frame of reference for the problem is inertial. However the frame of reference for the structure as a whole may have a nominal motion which is a specified constant velocity, acceleration, and angular velocity relative to the inertial frame of reference. Spin-up is not currently supported in GRASP. The frame for the structure as a whole is taken to be coincident with the inertial frame initially (since the position and orientation of the inertial frame are not relevant to solving the problem).

In addition, there is a reference frame associated with each substructure contained in the model hierarchy. These substructure frames of reference are selected to participate in the gross motion of the substructures. During calculations for a substructure, the inertial position and orientation of the superordinate (parent) frame and the position and orientation of the current frame relative to the superordinate frame are available. Given these, the inertial position and orientation of the current frame can be determined by using the appropriate chain rules as

$$\begin{aligned} \underline{R}^{FI} &= \underline{R}^{FS} + \underline{R}^{SI} \\ C^{FI} &= C^{FS}C^{SI} \end{aligned} \tag{1}$$

Each frame of reference, including the one for the whole structure, introduces six redundant generalized coordinates associated with the rigid-body motions of the frame. For the steady-state problem, the translational generalized coordinates are the measure numbers for the steady-state position of the frame relative to the reference position of the frame expressed in the basis associated with the steady-state position of the frame, $R_{F'_i}^{F'F}$. The rotational generalized coordinates are the Euler-Rodrigues parameters (also called Rodrigues parameters or components of the Rodrigues vector) for the steady-state orientation of the frame relative to the reference orientation of the frame, $\phi_i^{F'F}$.

GRASP's definition of Euler-Rodrigues parameters differs from the usual definition[13] by a factor of 2, *viz.*,

$$\phi_i^{BA} = 2\hat{e}_{A_i} \tan\frac{\theta}{2} \tag{2}$$

where a rotation of magnitude θ occurs about a unit vector $\hat{\underline{e}}$ (an Euler rotation) which carries the frame A into coincidence with frame B. The direction cosine matrix associated with an arbitrary set of those Euler-Rodrigues parameters arranged as a column matrix denoted by ϕ is

$$C^{BA} = \frac{(1 - \phi^T\phi/4)\Delta + \phi\phi^T/2 - \tilde{\phi}}{1 + \phi^T\phi/4} \tag{3}$$

The tilde symbol is defined by

$$\tilde{A}_{ij} = -\epsilon_{ijk}A_k \tag{4}$$

(Recall that the Levi-Civita symbol, ϵ_{ijk}, is 1 if i, j, k is an even permutation of $1, 2, 3$; -1 if i, j, k is an odd permutation of $1, 2, 3$; and 0 otherwise.) Besides the simplicity of the conversion to and from direction cosines, Euler-Rodrigues parameters have the advantage of being free of singularities for rotations up to 180°.

The generalized coordinates for the eigenproblem represent a perturbation beyond the steady-state values. The translational generalized coordinates are $\breve{R}_{F''_i}^{F''F'}$. Since the perturbations are infinitesimal, the rotational generalized coordinates can be associated with the measure numbers of a rotation vector $\breve{\theta}_{F''_i}^{F''F'}$. The direction cosine matrix associated with these generalized coordinates is

$$C^{F''F'} = \Delta - \tilde{\breve{\theta}}_{F''}^{F''F'} \tag{5}$$

To eliminate the redundant generalized coordinates and establish the position and motion of the frame relative to the substructure, a constraint must be specified. In GRASP, the constraint library currently contains three frame constraints: the fixed frame, the periodic frame, and the rotating frame constraints. The prescribed constraint, normally used to constrain nodal generalized coordinates, can also be used to prescribe specific frame generalized coordinates. Each of the frame constraints specifies the motion of the frame in terms of the superordinate (parent) frame.

When GRASP begins the calculations associated with a substructure, the first thing that is done is to calculate

the position and orientation of the frame of reference relative to the superordinate frame and the steady-state inertial motion of the frame of reference. The relative position and orientation, $\underline{R}^{F'S'}$ and $C^{F'S'}$, may be generated from the frame constraint parameters and the frame generalized coordinates for the current and superordinate frames. The steady-state inertial motion of the current frame of reference can be obtained from the same information as

$$\begin{aligned} \Omega^{F'I} &= \Omega^{F'S'} + \Omega^{S'I} \\ \underline{V}^{F'I} &= \Omega^{S'I} \times \underline{R}^{F'S'} + \underline{V}^{S'I} \\ \underline{A}^{F'I} &= \Omega^{S'I} \times \left(\Omega^{S'I} \times \underline{R}^{F'S'}\right) + \underline{A}^{S'I} \end{aligned} \qquad (6)$$

To obtain the contribution of frame generalized coordinates to the virtual work, virtual displacements of the frame generalized coordinates are required. The virtual displacements are just variations of the steady-state problem translational and the eigenproblem translational generalized coordinates. The virtual work associated with the steady-state problem rotational and the eigenproblem rotational generalized coordinates is treated differently. The virtual work is the dot product of the moment and a virtual rotation vector, the measure numbers of which are $\delta\psi_{F_i}^{F'F}$ and $\delta\psi_{F_i}^{F''F'}$ for the steady-state problem and eigenproblem, respectively. More detail may be found in Ref. 10 or the related discussion in a companion paper.[11]

Nodes

Substructure equations of motion in GRASP are developed based on a finite element formulation. Rather than solving for the field itself, nodes are used to define generalized coordinates that approximate the field in the vicinity of the node. In GRASP, there are two fields, the structural deformation field and the air flow field. The library of nodes supports both fields, containing one node for each field: the air node and the structural node. Air nodes to define the air flow fields in the aeroelastic beam and air mass elements. Structural nodes define the displacement fields in the aeroelastic beam and rigid-body mass elements.

Air Node

The only air flow field currently supported in GRASP is the axisymmetric flow field of a helicopter rotor including dynamic inflow effects.[14] The air node must be inertially fixed and located on the axis of symmetry. The basis associated with the air node, A, has the $\hat{b}_{A_1}$ axis pointed upwind along the axis of symmetry (vertical in hovering flight). The inertial velocity of the air flow, U, at any point, P, in the flow field is

$$\underline{U}^{PI} = -\left(q_1^A + R_\perp^{PA} q_2^A + R_{A_2}^{PA} q_3^A + R_{A_3}^{PA} q_4^A\right)\hat{b}_{A_1} \qquad (7)$$

where $R_\perp^{PA}$ is the perpendicular distance to the axis of the flow field

$$R_\perp^{PA} = \sqrt{(R_{A_2}^{PA})^2 + (R_{A_3}^{PA})^2} \qquad (8)$$

The four q_i^A are the air node generalized coordinates. q_1^A is the axial flow velocity which is an active generalized coordinate for both the steady-state problem and the eigenproblem. q_2^A is the radial velocity gradient which is active in the steady-state problem only (and is constrained to be zero in the eigenproblem). q_3^A and q_4^A are the cyclic velocity gradients which are both active only in the eigenproblem (and are constrained to be zero in the steady-state problem). For the eigenproblem, since the generalized coordinates are velocities and spatial gradients thereof, they are actually considered to be the time derivatives of generalized coordinates.

Structural Node

GRASP is designed to support displacement fields which are characterized by small strains but large displacements and rotations. Displacements are represented by the traditional three translational generalized coordinates. However, the usual approach of taking small rotations about the basis vectors as generalized coordinates is inappropriate for representing large rotations. Instead, the three generalized coordinates are taken to be the Euler-Rodrigues parameters which specify the orientation of the node after deformation. Thus the generalized coordinates for a structural node are the ones that would be appropriate for specifying the motion of an infinitesimal, massless rigid body that occupies the material point associated with the node.

The generalized coordinates for a structural node are, thus, analogous to the generalized coordinates for a frame of reference. The translational generalized coordinates for the steady-state problem and eigenproblem are $R_{N_i}^{N'N}$ and $\hat{R}_{N_i}^{N''N'}$. The rotational generalized coordinates for the steady-state problem and the eigenproblem are $\phi_i^{N'N}$ and $\hat{\theta}_{N_i}^{N''N'}$. For the translational generalized coordinates, the virtual displacements are just the variation of the translational generalized coordinates. For the rotational generalized coordinates, the virtual rotations $\delta\psi_{N_i}^{N'N}$ and $\delta\psi_{N_i}^{N''N'}$ are used in the steady-state problem and eigenproblem.

Elements

In its current version, GRASP includes three elements: the aeroelastic beam, the air mass, and the rigid-body mass. The equations for each of these elements were derived by using the principle of virtual work. After a general description of how one goes about deriving the equations for a general element, the derivation of the equations for a specific element, the aeroelastic beam, will be discussed.

Element Equations

In general, the derivation of the equation for a finite element involves first selecting a set of generalized coordinates that discretize the internal displacements of the element. Since the element displacements are also represented by nodal coordinates, a transformation between the nodal coordinates and element generalized coordinates must be defined. In the case of the rigid-body mass element, these two steps are trivial, since the nodal coordinates are identical to the element generalized coordinates.

Next, expressions for the virtual work of internal, body, and surface forces are derived. The internal forces arise from deformations of the element itself, while the body and surface forces are imposed on the element form outside sources. The rigid-body mass element, for example, is subject only to inertial and gravitational body forces.

Aeroelastic Beam Element

As an example of the derivation of the equations for an element, consider the aeroelastic beam. It is the most general element currently included in the GRASP element library, and is used to model slender beams undergoing small strains and large rotations. In the derivation, shear deformation and inplane distortion of the cross section are ignored. While warping rigidity is also ignored, all other effects of warping have been retained. The nonlinear beam kinematics for this type of element have been formulated and applied to the dynamic analysis of a pretwisted, rotating, beam element in Ref. 15.

In GRASP, the generalized coordinates of the aeroelastic beam element, Fig. 1, are carried by a frame of reference F, two structural nodes R and T, and an air node A. The axes of both the frame and the root structural node are centered at the shear center of the root-end cross section, and oriented along the cross-section principal axes. Similarly, the tip structural-node axes are centered at the shear center of the tip-end cross section, and oriented along the cross-section principal axes. The air node axes are centered at the flow field axis of symmetry, and oriented such the 1-axis is along the axis of symmetry.

Spatial Discretization

The interior displacements of the beam are represented by four functions of the axial coordinate x_3: u_i and θ_3. Bending is described by u_1 and u_2, axial displacement by u_3, and torsion by θ_3. These functions are discretized in terms of standard cubic and linear polynomials so that the generalized coordinates at the root and tip of the beam can be related to the nodal displacements and rotations. In addition, however, there are also generalized coordinates, termed internal degrees of freedom, associated with higher-order polynomials.

The variables u_i and θ_3 are expanded in a set of polynomials based on Ref. 16. The "C0" functions (u_3 and θ_3) are expanded in terms of $\psi_i(x)$ where $x = x_3/\ell$. All functions after the first two standard linear functions are orthonormalised. The "C1" functions (u_α) are expanded in terms of $\beta_i(x)$. Similarly, all functions after the first four standard cubic functions are orthonormalized.

Transformation of Coordinates

Since the nodes have a different set of generalized coordinates than those of the beam element, it is necessary to calculate the beam generalized coordinates in terms of the nodal displacements and rotational variables at both the root and tip of the beam. In general, the beam generalized coordinates are functions of the nodal displacements, and the orientation of the tip node relative to the root node.

In addition to the transformation of the nodal coordinates to beam generalized coordinates, the generalized forces associated the beam generalized coordinates must be transformed into the forces and moments at the root and tip nodes. In the steady-state problem, the transformation is straightforward, but tedious. However, in the dynamics the transformation contributes additional geometric stiffness terms at both the root and tip of the beam. These terms arise because the transformation equations, which must also perturbed for the dynamics, depend on the nodal coordinates. Geometric stiffness is also discussed below in reference to constraints. Further information and details of this calculation may be found in Ref. 10.

Elasticity

The elastic equations for the beam are based on the variation of the strain energy

$$\delta U = \int_0^\ell \iint_A (G\epsilon_{3\alpha}\delta\epsilon_{3\alpha} + E\epsilon_{33}\delta\epsilon_{33})\, d\xi_1 d\xi_2 dx_3 \tag{9}$$

where

$$\begin{aligned}
\epsilon_{31} &= (\lambda_1 - \xi_2)(\kappa_3 - \theta') \\
\epsilon_{32} &= (\lambda_2 + \xi_1)(\kappa_3 - \theta') \\
\epsilon_{33} &= \bar{\epsilon}_{33} + \xi_2\kappa_1 - \xi_1\kappa_2 \\
&\quad + \frac{1}{2}(\xi_1^2 + \xi_2^2)(\kappa_3 - \theta')^2 \\
&\quad + (\xi_2\lambda_1 - \xi_1\lambda_2)\theta'(\kappa_3 - \theta')
\end{aligned} \tag{10}$$

and where $\theta(x_3)$ is the pretwist angle, with $\theta(0) = 0$, and $(\ )' = d(\ )/dx_3$. The components of the curvature vector, κ_i, depend nonlinearly on u'_α and θ_3, and the extension of the elastic axis, $\bar{\epsilon}_{33}$, depends nonlinearly on u'_i. After integrating over the cross sectional area, we obtain the variation of the strain energy as follows:

$$\delta U = \int_0^\ell (F_3\delta\bar{\epsilon}_{33} + M_1\delta\kappa_1 + M_2\delta\kappa_2 + M_3\delta\kappa_3)\, dx_3 \tag{11}$$

where F_3, M_1, M_2, and M_3 are the stress resultants, which can be expressed in terms of $\bar{\epsilon}_{33}$ and κ_i. Details of this derivation can be found in Ref. 15.

Inertial/Gravitational

The generalized forces due to the motion of the aeroelastic beam relative to an inertial frame are also derived following Ref. 15. This portion of the derivation, which ignores warping, is based on the work done by inertial and gravitational forces acting through a virtual displacement. The virtual work is then

$$\begin{aligned}
\delta W &= \int_0^\ell (\delta\underline{u}^P \cdot \underline{F}^P + \underline{\delta\psi}^P \cdot \underline{M}^P)dx_3 \\
&\quad + \delta\underline{u}^F \cdot \int_0^\ell \underline{F}^P dx_3 + \underline{\delta\psi}^F \int_0^\ell (\underline{M}^P + \underline{R}^{PF} \times \underline{F}^P)dx_3
\end{aligned} \tag{12}$$

where $\underline{F}^P$ and $\underline{M}^P$ are, respectively, the inertial and gravitational forces and moments associated with a generic point P along the beam axis, $\underline{R}^{PF}$ is the position of that point relative to the element frame, $\delta\underline{u}^P$ and $\underline{\delta\psi}^P$ are the virtual displacement and virtual rotation at that point, and $\delta\underline{u}^F$ and $\underline{\delta\psi}^F$ are the virtual displacement and virtual rotation of the frame.

Aerodynamics

The aerodynamic forces acting on the aeroelastic beam element are determined from a quasi-steady adaptation of Greenberg's thin-airfoil theory (Ref. 17). Two new sets of coordinate axes must be introduced for the purpose of defining the directions in which the aerodynamic forces and pitching moment act. In Fig. 2, the Z axes are shown to be a set of dextral axes associated with the zero-lift line for the airfoil section. The other set of axes is the so-called wind axes W. For these axes the base vector $\hat{b}_{W_3}$ is identical to $\hat{b}_{Z_3}$. The base vector $\hat{b}_{W_2}$ is along the relative wind vector (in the direction of drag) and $\hat{b}_{W_1}$ is in the direction of lift. As with other axes, these axes convect with the local beam cross section during deformation.

The aerodynamic forces and pitching moment act at the aerodynamic center Q (which is assumed to be the quarter chord). The offset position of Q relative to the origin of the local principal axes P is $R^{QP}_{Z_2}\hat{b}_{Z_2}$. The dynamic wind velocity vector at the aerodynamic center denoted by $\underline{W}^{Q''}$ is calculated by subtracting the inertial structural velocity at Q'' from the inertial air velocity at Q''.

The angle of attack is determined from

$$\tan\alpha = \frac{W^{Q''}_{Z''_1}}{W^{Q''}_{Z''_2}} \tag{13}$$

The local air flow velocity gradient is expressed as

$$G^{Q''}_{Z''_{13}} = \frac{\partial W^{Q''}_{Z''_1}}{\partial R^{QP}_{Z_2}} \tag{14}$$

The relative wind velocity magnitude is given by

$$W = \sqrt{\left(W^{Q''}_{Z''_1}\right)^2 + \left(W^{Q''}_{Z''_2}\right)^2} \tag{15}$$

Both the relative wind velocity and the angle of attack are time-dependent quantities that depend on the kinematical variables for the aeroelastic beam element, including frame motion and induced inflow velocity degrees of freedom. The static and dynamic perturbations of these quantities are determined and used in GRASP in their exact form to calculate the generalized forces.

The applied force on the beam is assumed to be

$$\underline{F} = \mathcal{L}_c\hat{b}_{W''_1} + \mathcal{D}\hat{b}_{W''_2} + \mathcal{L}_{nc}\hat{b}_{Z''_1} \tag{16}$$

and the applied moment is

$$\underline{M} = \mathcal{M}\hat{b}_{Z''_3} \tag{17}$$

The governing equations for the components of the aerodynamic forces and moments are adapted from thin airfoil theory with arbitrary lift, drag and pitching moment coefficients c_l, c_d, and c_m.

$$\begin{aligned}
\mathcal{L}_c &= \frac{1}{2}\rho_a W^2 c c_l + \frac{\pi}{2}\rho_a c^2 W G^{Q''}_{Z''_{13}} \\
\mathcal{D} &= \frac{1}{2}\rho_a W^2 c c_d \\
\mathcal{M} &= \frac{1}{2}\rho_a W^2 c^2 c_m - \frac{\pi}{16}\rho_a c^3 \Big(W G^{Q''}_{Z''_{13}} \\
&\qquad + \dot{W}^{Q''}_{Z''_1} + \frac{3c}{8}\dot{G}^{Q''}_{Z''_{13}}\Big) \\
\mathcal{L}_{nc} &= \frac{\pi}{4}\rho_a c^2 \left(\dot{W}^{Q''}_{Z''_1} + \frac{c}{4}\dot{G}^{Q''}_{Z''_{13}}\right)
\end{aligned} \tag{18}$$

With the assumption that the air exerts a force on the structure that is equal and opposite to that of the structure on the air, the virtual work for a dynamic perturbation can be formed as

$$\delta W = \int_0^\ell \left(-\delta S^{Q''}_{Z''_\beta} F_{Z''_\beta} + \delta \Upsilon^{Q''}_{Z''_3}\mathcal{M}\right) dx_3 \tag{19}$$

where the $\delta S^{Q''}_{Z''_\beta}$ and $\delta\Upsilon^{Q''}_{Z''_3}$ are the virtual displacements and rotation of an element of air relative to the structure, respectively. These virtual displacements and rotations include all of the kinematical variables for the beam element as well as degrees of freedom representing frame motion and dynamic inflow velocity and velocity gradients. The virtual work per unit of beam element length done by the aerodynamic forces and pitching moment can then be put into the form of steady-state generalized forces and linear coefficient matrices for generalized accelerations, velocities, and displacements. Details of calculating the aerodynamic contribution to the elements of M, C, K, and $\hat{Q}$ for the discretized system and other aspects of the formulation can be found in Ref. 10. It is worthwhile to note that the aerodynamic contribution to the matrix M is symmetric.

Constraints

The GRASP constraint library contains constraints serving a variety of purposes. These include frame constraints, element connectivity constraints, nodal and generalized coordinate constraints, and constraints generated by GRASP that remain internal to the program (*e.g.*, the node promotion/demotion constraints mentioned in the substructuring section). All of the constraints which are currently in the library have certain features in common. This permits a general description of the way in which constraints are treated. The treatment for a specific constraint, the screw, is then presented as an illustration. The screw constraint highlights the important role of geometric stiffness in dynamical analyses with nonlinear constraints. Further detail on the screw and other constraints can be found in Ref. 10.

Treatment of Constraints in General

A constraint creates dependency among the generalized coordinates. In GRASP, the constraints and the dependency amongst the generalized coordinates are eliminated so that the actual matrix eigenproblem and the iteration for a steady-state solution are performed only on the independent problem degrees of freedom. For all of the constraints currently in the GRASP library, the constraint relationship can be written in the special form

$$q_{e_i} = g_i\left(q_{r_1}, \ldots, q_{r_{N^r}}\right), \qquad (i = 1, \ldots, N^e) \tag{20}$$

Thus the generalized coordinates related to the constraint can be partitioned into two sets. And, the set to be eliminated can be obtained directly from the constraint functions which only depend on the set to be retained. All of a substructure's generalized coordinates can be obtained, using the constraint relationships, to obtain the eliminated generalized coordinates from the retained generalized coordinates. (It was noted in the substructuring section that in the case of the steady-state problem state vector, the independent degrees of freedom of the substructure are stored in the parent substructure.)

The virtual work for the generalized coordinates associated with the constraint is

$$\delta W = \sum_{i=1}^{N^e} \delta q_{e_i} Q_{e_i} + \sum_{i=1}^{N^r} \delta q_{r_i} Q_{r_i} \tag{21}$$

The sum of the generalized forces associated with a generalized coordinate, Q, may differ from zero for two reasons. First, while seeking a steady-state solution, equilibrium is not satisfied and these residuals are a measure of the error in the approximate solution. Second, even if the system is in equilibrium, individual substructures will not be. For instance, the root node for an aeroelastic beam element will not be in equilibrium at the element level substructure; it will only be in equilibrium in a higher level substructure where the contributions from all the elements connected to that node have been included.

Taking the variation of Eq. (20) yields

$$\delta q_{e_i} = \sum_{j=1}^{N^r} \frac{\partial g_i}{\partial q_{r_j}} \delta q_{r_j} \qquad (i = 1, \ldots, N^e) \tag{22}$$

Substituting Eq. (22) into Eq. (21) yields

$$\delta W = \sum_{j=1}^{N^r} \delta q_{r_j} \left(Q_{r_j} + \sum_{i=1}^{N^e} \frac{\partial g_i}{\partial q_{r_j}} Q_{e_i} \right) \tag{23}$$

This relationship is used by GRASP to incorporate the contributions of the generalized forces associated with the eliminated generalized coordinates into the retained generalized forces. During calculation of steady-state residuals, the residuals associated with the eliminated generalized coordinates are transformed and added to the appropriate residuals in the parent substructure's residual vector.

The treatment of constraints for the eigenproblem is a little more involved. For the eigenproblem, the generalized coordinates are assumed to be the sum of a steady-state value and an infinitesimal perturbation (*i.e.*, $q = \bar{q} + \hat{q}$). Equations (20), (22), and the generalized forces, Q can all be expanded in a Taylor series about the steady-state value. Noting that Eq. (20) is valid when $q = \bar{q}$, expansion of Eq. (20) yields

$$\hat{q}_{e_i} = \sum_{j=1}^{N^r} \overline{\frac{\partial g_i}{\partial q_{r_j}}} \hat{q}_{r_j} + \cdots \qquad (i = 1, \ldots, N^e) \tag{24}$$

Expansion of Eq. (22) yields

$$\delta q_{e_i} = \sum_{j=1}^{N^r} \delta q_{r_j} \left(\overline{\frac{\partial g_i}{\partial q_{r_j}}} + \sum_{h=1}^{N^r} \overline{\frac{\partial^2 g_i}{\partial q_{r_j} \partial q_{r_h}}} \hat{q}_{r_h} + \cdots \right) \qquad (i = 1, \ldots, N^e) \tag{25}$$

Expansion of the generalized force, Q, including both eliminated and retained terms, yields

$$Q_i = \overline{Q}_i + \sum_{j=1}^{N} \overline{L}_{ij} \hat{q}_j \qquad (i = 1, \ldots, N) \tag{26}$$

where the linear operator, $\overline{L}$, contains the terms normally associated with the mass, damping and stiffness matrices, $-M\frac{d^2}{dt^2} - C\frac{d}{dt} - K$. (The minus signs are present because the generalized force is generally regarded as positive on the right hand side of the dynamical equation, whereas the linear coefficient matrices are regarded as positive on the left hand side.)

GRASP calculates the M, C, and K matrices for a substructure by adding the contributions of each of its children. The rows and columns of the child substructure's matrices correspond to all of the generalized coordinates of the child. The constraints are used to eliminate dependent generalized coordinates, resulting in matrices whose rows and columns correspond to only the retained generalized coordinates of the child. The matrices elements are then added to the elements of the parent substructure's matrices that correspond to the child's independent degrees of freedom. The required transformations can be found using the virtual work for the substructure.

An expression of virtual work expanded about the steady-state value may be obtained by substituting Eq. (25) and the eliminated and retained subsets of Eq. (26) into the virtual work expression in Eq. (21). After discarding terms of second or higher order, the result is a constant term and two first order terms in $\hat{q}$. The constant, first term is the same as Eq. (23) evaluated at the steady-state equilibrium.

The linear, second term is

$$\begin{aligned} &\sum_{j=1}^{N^r} \sum_{h=1}^{N^r} \delta q_{r_j} \left(\sum_{i=1}^{N^e} \overline{\frac{\partial^2 g_i}{\partial q_{r_j} \partial q_{r_h}}} \overline{Q}_{e_i} \right) \hat{q}_{r_h} \\ &= \sum_{j=1}^{N^r} \sum_{h=1}^{N^r} \delta q_{r_j} \left(-K^G_{r_j r_h} \right) \hat{q}_{r_h} \end{aligned} \tag{27}$$

The matrix K^G represents the geometric stiffness associated with the constraint. During assembly of the matrices for the parent substructure, GRASP calculates this geometric stiffness and adds it to the stiffness matrix in the parent substructure. This extremely important term is often overlooked. For instance, a pendulum, modeled as a rigid-body mass constrained to rotate about an offset axis (using a screw constraint) derives *all* of its stiffness from this geometric stiffness term.

The linear, third term is

$$\sum_{j=1}^{N^r} \sum_{h=1}^{N} \delta q_{r_j} \left(\overline{L}_{r_j h} + \sum_{i=1}^{N^e} \overline{\frac{\partial g_i}{\partial q_{r_j}}} \overline{L}_{e_i h} \right) \hat{q}_h \tag{28}$$

Separating the sum on k into partial sums involving only eliminated or retained generalized coordinates, and substituting Eq. (24) into Eq. (28) for the eliminated perturbation coordinates yields for the third term

$$\sum_{j=1}^{N^r}\sum_{h=1}^{N^r}\delta q_{r_j}\left(\overline{L}_{r_j r_h}+\sum_{i=1}^{N^e}\overline{\frac{\partial g_i}{\partial q_{r_j}}}\,\overline{L}_{e_i r_h}+\sum_{l=1}^{N^e}\overline{L}_{r_j e_l}\overline{\frac{\partial g_l}{\partial q_{r_h}}}\right.$$
$$\left.+\sum_{i=1}^{N^e}\sum_{l=1}^{N^e}\overline{\frac{\partial g_i}{\partial q_{r_j}}}\,\overline{L}_{e_i e_l}\overline{\frac{\partial g_l}{\partial q_{r_h}}}\right)\ddot{q}_{r_h} \tag{29}$$

The quantity within the parentheses in Eq. (29) can be thought of as defining a new set of M, C, and K matrices in terms of the retained and eliminated portions of the original matrices. GRASP calculates the new matrices and adds their elements to the elements of the parent substructure's matrices.

The complete definition of a constraint, then, follows from specification of g. An understanding of the effect of the constraint in the dynamics, however, cannot be obtained without explicit calculation of the matrix $\frac{\partial g}{\partial q}$ and the geometric stiffness matrix K^G.

Example Constraint: The Screw

For the screw constraint, consider two nodes connected by a mechanism that permits translation and rotation. Specifically, one node is permitted to displace relative to the other along an axis, which is denoted as the screw axis, that is fixed in the deflected and rotated coordinate system of both nodes. Also, one node is permitted to rotate relative to the other about that same axis. Denote the dependent node (whose generalized coordinates are to be eliminated) by D and the independent node (whose generalized coordinates are to be retained) by I. The I generalized coordinates are known and it is necessary to write a relationship between them and the D generalized coordinates. The screw axis is identified by a unit vector $\hat{\underline{e}}^{scr}$. To simplify the algebra, two intermediate nodes are introduced denoted by S and M (for "stationary" and "moving"), located on the screw axis. I and S are locked together during deformation of the substructures to which they are attached as are D and M. M and S initially coincide positionally but may differ in orientation. R_I^{DI}, C^{DI}, R_D^{DM}, and R_I^{SI} are assumed to be given.

Statics

For the static problem, the relationship governing the generalized coordinates and the contribution of the force and moment acting at D' to those at I' must be written. The basic displacement and orientation relationships govern the position and orientation of the dependent node in terms of those for the independent node and are

$$\underline{R}^{D'D}=\underline{R}^{D'M'}+\underline{R}^{M'S'}+\underline{R}^{S'I'}+\underline{R}^{I'I}$$
$$+\underline{R}^{IS}+\underline{R}^{SM}+\underline{R}^{MD} \tag{30}$$
$$C^{D'D}=C^{D'M'}C^{M'\tilde{M}'}C^{\tilde{M}'S'}C^{S'I'}C^{I'I}C^{ID}$$

where $\tilde{M}'$ denotes a node whose position and orientation relative to S' is the same as that of M relative to S. In order to obtain the component form of the kinematical relation corresponding to the function g above, the first of Eqs. (30) are written in component form. Let $\underline{R}^{M'S'}=u\hat{\underline{e}}^{scr'}$ where u is the screw displacement and $\underline{R}^{MS}=0$. In the D basis, the position becomes

$$R_D^{D'D}=C^{DI}C^{II'}C^{I'S'}C^{S'\tilde{M}'}C^{\tilde{M}'M'}C^{M'D'}R_{D'}^{D'M'}$$
$$-R_D^{DM}+C^{DI}C^{II'}C^{I'S'}ue_{S'}^{scr'} \tag{31}$$
$$+C^{DI}(C^{II'}R_{I'}^{S'I'}+R_I^{I'I}-R_I^{SI})$$

The equations simplify somewhat if $C^{IS}=C^{I'S'}=C^{D'M'}=C^{DM}=\Delta$. $C^{M'\tilde{M}'}$ is nicely expressed as an Euler rotation given θ, the screw rotation, and the unit vector about which the rotation occurs is $e_{S'}^{scr'}=e_{\tilde{M}'}^{scr'}$, the screw axis unit vector. The second of Eqs. (30) is implemented in GRASP so that first each of the direction cosine matrices on the right hand side is determined. All are given in terms of geometrical parameters for the screw except $C^{M'\tilde{M}'}$ which is known in terms of the screw rotation angle θ and $C^{I'I}$ which is known in terms of the Euler-Rodrigues parameters for the node I. The Euler-Rodrigues parameters at D can then be calculated once $C^{D'D}$ is known. To write out this nonlinear transformation in detail would be quite difficult, but part of the philosophy in GRASP is that such equations are *not* to be written out in explicit form. This reduces the chance for error and makes the code more readable.

For the purpose of transforming the generalized forces, *which for this constraint are components of the actual force* and moment at I' and the screw force and moment, the virtual displacement and virtual rotation components must be determined. The virtual displacement components are the same as the Lagrangian variation of the vector components, namely

$$\delta R_D^{D'D}=C^{DI}\big[\delta R_I^{I'I}-(\tilde{R}_I^{D'M'}+u\tilde{e}_I^{scr'}+\tilde{R}_I^{S'I'})\delta\psi_I^{I'I}$$
$$+e_I^{scr'}\delta u+\tilde{e}_I^{scr'}R_I^{D'M'}\delta\theta\big] \tag{32}$$

The virtual rotation components are given by

$$\delta\psi_D^{D'D}=C^{DI}(\delta\psi_I^{I'I}+e_I^{scr'}\delta\theta) \tag{33}$$

A force and moment at D' will contribute to the virtual work at the screw connection (through δu and $\delta\theta$) and at I' (through $\delta R_I^{I'I}$ and $\delta\psi_I^{I'I}$). The virtual work done by $\underline{F}^{D'}$ and $\underline{M}^{D'}$ is

$$\delta W=(\delta R_I^{I'I})^T F_I^{D'}+(\delta\psi_I^{I'I})^T(\tilde{R}_I^{D'I'}F_I^{D'}+M_I^{D'})$$
$$+\delta u(e_I^{scr'})^T F_I^{D'} \tag{34}$$
$$+\delta\theta(e_I^{scr'})^T(\tilde{R}_I^{D'M'}F_I^{D'}+M_I^{D'})$$

Clearly, the force components at I' are just those at D' transformed into the I basis. The moment components at I', however, include not only the moment at D' transformed into the I basis, but also, as expected, the moment of the force at D' about I' in the I basis. The screw force is the component of the force at D' along the screw axis for

the deformed structure. Similarly, the screw moment is the component of the moment at D' along the screw axis, augmented by the moment of the force at D' about the screw axis.

The $\frac{\partial s}{\partial q}$ matrix turns out to be

$$\begin{bmatrix} C^{DI} & -C^{DI}\tilde{R}_I^{D'I'} & C^{DI}e_I^{scr'} & -C^{DI}\tilde{R}_I^{D'M'}e_I^{scr'} \\ 0 & C^{DI} & 0 & C^{DI}e_I^{scr'} \end{bmatrix} \tag{35}$$

where the columns of the matrix are associated with the variations of the generalized coordinates $\delta R_I^{I'I}$, $\delta\psi_I^{I'I}$, δu, and $\delta\theta$ and the rows of the matrix are associated with $\delta R_D^{D'D}$ and $\delta\psi_D^{D'D}$.

Dynamics

For the dynamics problem, a similar relationship governing the generalized coordinates is used to find the matrices $\frac{\partial s}{\partial q}$ and K^G. Consider the nodes and the screw axis in their dynamic states, an infinitesimal perturbation from the static position and orientation. The basic equations for the position are similar to those of the static case

$$\begin{aligned} \underline{R}^{D''D'} &= \underline{R}^{D''M''} + \underline{R}^{M''\bar{M}''} + \underline{R}^{\bar{M}''S''} \\ &\quad + \underline{R}^{S''I''} + \underline{R}^{I''I'} + \underline{R}^{I'D'} \end{aligned} \tag{36}$$

For the orientation, as an alternative to the above, the virtual rotations can be used directly.

$$\begin{aligned} \underline{\delta\psi}^{D''D'} &= \underline{\delta\psi}^{D''M''} + \underline{\delta\psi}^{M''\bar{M}''} + \underline{\delta\psi}^{\bar{M}''S''} \\ &\quad + \underline{\delta\psi}^{S''I''} + \underline{\delta\psi}^{I''S'} + \underline{\delta\psi}^{I'D'} \end{aligned} \tag{37}$$

Here the first, third, fourth, and sixth terms are zero. Proceeding as above and noting that

$$\begin{aligned} C^{I''I'} &= \Delta - C^{I'I}\tilde{\theta}_I^{I''I'}C^{II'} = C^{I'I}(\Delta - \tilde{\theta}_I^{I''I'})C^{II'} \\ \delta C^{I''I'} &= -C^{I'I}(\Delta - \tilde{\theta}_I^{I''I'})\widetilde{\delta\psi}_I^{I''I'}C^{II'} \\ \delta C^{M''\bar{M}''} &= -\delta\theta\tilde{e}_{M''}^{scr''}C^{M''\bar{M}''} \end{aligned} \tag{38}$$

the $\frac{\partial s}{\partial q}$ matrix turns out to be the same as Eq. (35). It should be noted, however, that with a different choice of coordinate bases for the nodes, such as is used for the frames in GRASP,[10] the matrix $\frac{\partial s}{\partial q}$ may not be the same for statics as it is in dynamics. The geometric stiffness matrix K^G turns out to be

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -\tilde{F}_I^{D'}\tilde{R}_I^{D'I'} & \tilde{F}_I^{D'}e_I^{scr'} & -\tilde{F}_I^{D'}\tilde{R}_I^{D'M'}e_I^{scr'} \\ 0 & -(e_I^{scr'})^T\tilde{F}_I^{D'} & 0 & 0 \\ 0 & \begin{matrix} -(e_I^{scr'})^T\tilde{R}_I^{D'M'}\tilde{F}_I^{D'} \\ -(e_I^{scr'})^T\tilde{M}_I^{D'} \end{matrix} & 0 & \begin{matrix} -(R_I^{D'M'})^T\tilde{e}_I^{scr'} \\ \tilde{e}_I^{scr'}F_I^{D'} \end{matrix} \end{bmatrix} \tag{39}$$

where the columns correspond to the perturbations of the generalized coordinates $\check{R}_I^{I''I'}$, $\check{\theta}_I^{I''I'}$, $\check{u}$, and $\check{\theta}$ and the rows correspond to the variations of the generalized coordinates $\delta R_I^{I''I'}$, $\delta\psi_I^{I''I'}$, $\delta\check{u}$, and $\delta\check{\theta}$.

Note that the geometric stiffness matrix is not necessarily symmetric. In particular, note the three-by-three rotation-rotation term associated with the independent node. It is

$$-(\delta\psi_I^{I''I'})^T\tilde{F}_I^{D'}\tilde{R}_I^{D'I'}\check{\theta}_I^{I''I'} \tag{40}$$

The reasons for this are the choices of generalized coordinates (infinitesimal rotations) and virtual generalized coordinates (virtual rotations) for the equations. A similar result was obtained by Roberson and Likins.[18] It is easily shown that, for example, use of Euler-Rodrigues parameters and their variations to express both quantities, respectively, leads to a symmetric geometric stiffness matrix. The present method was implemented in GRASP because rows of the matrix equations corresponding to the virtual rotations are physical moments whereas the generalized forces associated with the variation of Euler-Rodrigues parameters have no easily identifiable physical significance.

Just from Eqs. (39) and (40) it is evident that geometric stiffness is an important phenomenon. It must be accounted for under all circumstances in which there is a coordinate transformation that in nonlinear. Most frequently, this phenomenon is associated with finite rotation of the coordinate axes that are involved in the transformation.

Concluding Remarks

In response to the limitations of previous methods for analyzing rotorcraft stability, GRASP has been developed. The first version of GRASP provides both a nonlinear steady-state solution and linearized eigensolution for rotorcraft in axial flight and ground contact conditions. Time-varying solutions are not provided in the current version. GRASP is the first program to be written utilizing the new methodology described in a companion paper.[17] The methodology permits both discrete motions between portions of the structure and deformation of the structure. It thus combines the flexibility in structural representation available with the traditional finite element approach with the ability to handle large discrete motions between portions of the structure available with the multibody approach usually associated with spacecraft applications.

In GRASP, the first step of the methodology, the decomposition of the structure into substructures, has received additional emphasis. A hierarchical decomposition of the structure is supported and continues to the finite element level.

The second step of the methodology, associating a frame of reference with each substructure, is supported by a library of frame constraints. There must be a frame constraint to specify the behavior of the 6 redundant generalized coordinates introduced by the frame. The library currently supports frames at a constant relative displacement and orientation, frames rotating at a constant angular velocity, and frames replicated periodically about an axis. By adding frame constraints to the library, additional ways of associating a frame of reference with a substructure can be incorporated in GRASP.

The third step of the methodology, developing the equations of motion, is based on the finite element method in GRASP. GRASP contains libraries of nodes, finite elements, and constraints specifying the connectivity of the elements. Currently, there are two types of nodes: structural and air; and three types of elements and associated connectivity constraints: aeroelastic beam, air mass, and rigid-body mass.

By adding nodes and elements to the libraries, additional modeling capability can be incorporated in GRASP.

Two characteristics of the GRASP finite element models are noteworthy. First, they include terms reflecting the motion of the frame of reference for the substructure containing the element. Second they include nonlinear effects and support deformations which result in large displacements and rotations. The aeroelastic beam element was used to provide an example of the implementation of a finite element in GRASP. It provides a powerful modeling tool, allowing the user to select the order of the polynomial approximation of the displacement field and incorporating aeroelastic effects.

The fourth and final step in the methodology, the specification of constraints, is supported by a library of constraints. In addition to the previously mentioned frame and connectivity constraints, there are a collection of constraints which constrain nodal generalized coordinates. The library currently supports prescribing a specified generalized coordinate, rigidly connecting two nodes, and allowing screw motion (translation along and rotation about a single axis). By adding constraints to the library, additional ways of constraining the motion of nodes can be incorporated in GRASP.

One of the most powerful constraints is the screw constraint, which can be used to build up a variety of gimbal or hinged connections. Besides providing an example of the implementation of a constraint in GRASP, it demonstrates the importance of nonlinear effects resulting in geometric stiffness in the model.

The algorithms used for solving the equations obtained with the methodology were outlined in a companion paper.[11] Solutions obtained with the methodology have been excellent. Comparisons of GRASP results with experimental results in the nonlinear analysis of a cantilever beam are reported on in Refs. 9 and 15 and in a companion paper.[19] Ref. 20 reports on the correlation of GRASP results with experimental data for a torsionally-soft rotor.

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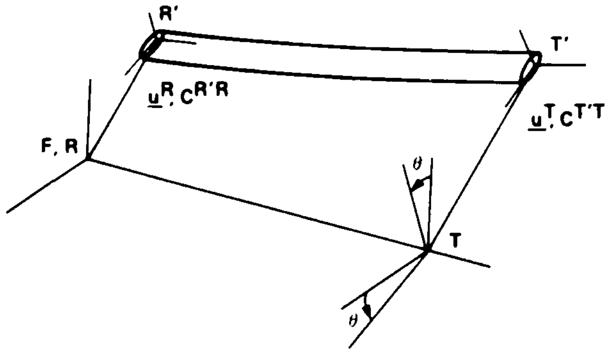

Fig. 1 Deformed positions and orientations of the aeroelastic beam element relative to its initial state.

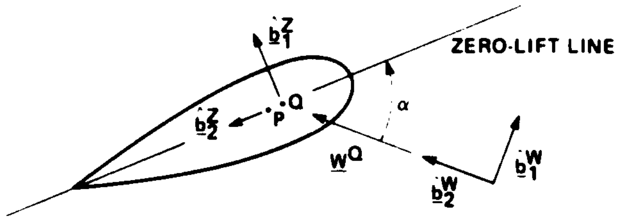

Fig. 2 Coordinate systems for the aeroelastic beam aerodynamics.

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